Advanced Mathematical Models & Applications
Vol.3, No.3, 2018, pp.211-214

# INVESTIGATION OF THE LOCAL NEURAL NETWORK NORMALIZED MODEL

**Valery A. Nikolsky***

Transport and Telecommunication Institute, Riga, Latvia

**Abstract.** In the present paper, the problem of a local neural network analytical model construction is considered. Model construction and investigation are based on stochastic normalization of the system including a neuron-transmitter and a neuron-receiver, interconnected by means of a communication link by way of axon. Relations are got for estimation of operating descriptions of a network in the stationary mode.

**Keywords**: simplest neural network, queuing system, the normalized model.
**AMS Subject Classification:** 92B20, 68T05, 91E40, 91E10, 90B15.
**Corresponding author:** Dr.Sc.Eng., As. Professor Valery A. Nikolsky, Transport and Telecommunication Institute, Computer Science and Electronics Faculty, 1 Lomonosov Street, Riga LV-1019, Latvia, Tel.: (+371)29595329, e-mail: *valerij.nikolsky@gmail.com*
*Received: 26 June 2018; Revised: 01 July 2018; Accepted: 10 July 2018; Published: 28 December 2018.*

## 1 Introduction

It is known that man's nerve tissue, including tissue of its brain, is sufficiently complicated. Therefore their cognition is conducted by neuroscience from the great number of directions (Nicholls et al., 2001), (Brockman, 2002). In given paper an elementary network consisting of two sequentially connected neurons is considered. The flow diagram of network is presented on the Fig. 1.

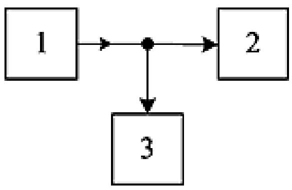


**Figure 1:** Two sequentially connected neurons

On the scheme the following denotations are accepted: 1 – a neuron-transmitter generating a stream of nervous signals (messages) with intensity $\lambda$; 2 – a neuron-receiver. The stream of occupied receiver releases has intensity $\mu$. Signal transmission is carried out along axon with intensity $\nu$ messages in time unit. The axon is divided into segments by Ranvier nodes (for instance, see (Virchow, 1854). At its end the axon branches on fibres. There are synapses on endings of these fibres. In the course of transmission the part of nervous signals for whatever reasons can abandon a network, not being serviced. It is reflected on Fig. 1 by rectangle with digit 3. Unserved signals on nervous fibres move to other neurons fulfilling the same functions, as a neuron-receiver. Therefore loss of the information in the absence of pathology does not happen.

Further the neural network we will interpret as a normalized single-channel queuing system (QS) (Fig. 2) (Nikolsky & Gamkrelidze, 1990). Then $X(t)$ it is possible to consider as a sequence of the nervous signals arriving to the system from a neuron-transmitter; $D$ is the distributor defining a direction of nervous signals movement depending on the state of the device DV (neuron-receiver) and presence of available spaces in a queue (communication link); $Z_i^*(t)$ is a stream of the messages bounding for communication link (queue) $i = \overline{1,k}$; $k$ is number of axon segments (places of presence in queue);$Z(t)$ is a stream of the signals arriving in the device when the system is free; $Y_1(t)$ is a stream of the served messages; $Y_2(t)$ is a stream of the messages abandoning network, not being serviced; $U(t)$ and $G_i(t)$ are signals representing device and queue state, respectively; $Y_1''(t)$ ,$Z_1''(t)$ ,${G_1}''(t)$ ,${U_1}''(t)$ are Gaussian noise system with zero expectations.

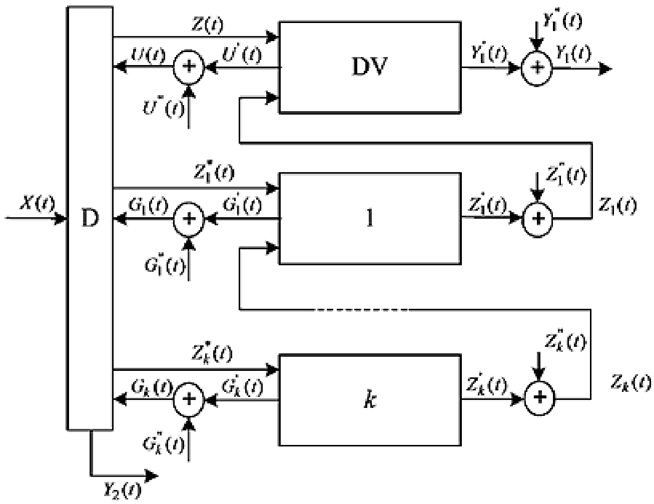

**Figure 2:** The image of the elementary neural network in the form of normalized single-channel QS

## 2 Mathematical description and investigation of the network model

The set of equations describing the system in dynamics looks like in the work (Nikolsky & Gamkrelidze, 1990). Considering rapidity of the transients in a neural network, we will limit our attention to reviewing of system behavior in a stationary mode. Then for the expectation values of state variables we will get

$$m_z = m_x \cdot (1 - m_u),$$

$$m_{y_2} = m_x \cdot m_{g_i},$$

$$m_{z_i^*} = m_x \cdot \left(m_{g_{i-1}} - m_g\right),$$

$$m_{z_i} = \left(m_{z_i^*} + m_{z_{i+1}}\right) \cdot \int_{-\infty}^{t} \varphi^-(t-\tau)\, d\tau,$$

$$m_{g_i} = \left(m_{z_i^*} + m_{z_{i+1}}\right) \cdot \int_{-\infty}^{t} \psi^-(t-\tau)\, d\tau,$$

$$m_{y_1} = (m_z + m_{z_1}) \cdot \int_{-\infty}^{t} \varphi(t-\tau)\, d\tau,$$

$$m_u = (m_z + m_{z_1}) \cdot \int_{-\infty}^{t} \psi(t-\tau)\, d\tau,$$

where $\varphi(t-\tau)$, $\psi(t-\tau)$, $\varphi^-(t-\tau)$, $\psi^-(t-\tau)$ are the weight functions received from the corresponding density functions of a holding time in the device and a response time in the queue, at that $\psi(\sigma) = \int_0^\sigma \varphi(t)\,dt$.

Let's enumerate possible system states: $S_0$ shows that the system is free; $S_1$ shows that the device is busy, a queue is absent; $S_2$ shows that the device is busy, one message in the queue; etc.; and finally, $S_{k+1}$ shows that the device (receiver) is busy, $k$ messages are in a communication link. To these states in the normalized model corresponds following probabilities:

$$P_0 = 1 - m_u,$$

$$P_1 = m_u - m_{g_1},$$

$$P_i = m_{g_{i-1}} - m_{g_i}, \ \ \forall i = \overline{2,\,k},$$

$$P_{k+1} = m_{g_k},$$

where it is assumed that the following conditions are satisfied:

$$\int_{-\infty}^{t} \varphi(t-\tau)\,d\tau = 1, \ \int_{-\infty}^{t} \varphi^-(t-\tau)\,d\tau = 1,$$

$$\int_{-\infty}^{t} \psi(t-\tau)\,d\tau = \frac{1}{\mu}, \ \int_{-\infty}^{t} \psi^-(t-\tau)\,d\tau = \frac{1}{\nu}.$$

Consequently, we will get

$$P_{k+1} = \frac{\rho \cdot \rho_1^k \cdot (1-\rho_1)}{1+\rho_1^k \cdot (\rho - \rho_1 - \rho \cdot \rho_1)},$$

$$P_0 = \frac{1-\rho+\rho_1^k \cdot (\rho - \rho_1)}{1+\rho_1^k \cdot (\rho - \rho_1 - \rho \cdot \rho_1)},$$

$$m_{y_1} = m_x \cdot (1 - P_{k+1}),$$
$$m_{y_2} = m_x \cdot P_{k+1},$$

where $P_{k+1}$ means the probability of refuse in service; $P_0$ denotes the probability of the system is free; $m_{y_1}$ is network absolute capacity; $m_{y_2}$ is an average number of messages abandoned network unserved; $\rho$ denotes the capacity factor of a neuron-receiver; $\rho_1$ means the capacity factor of the segment of communication link; $m_x = \lambda$ is an average speed of entering in axon of nervous signals from the neuron-transmitter.

Important indexes presenting practical interest are the average number and average sojourn time of messages in the communication link and the network.

The average number of messages in a communication link can be calculated by the formula:

$$\bar{r} = \sum_{i=2}^{k+1`} (i-1) \cdot P_i = \frac{\rho \cdot \rho_1 \cdot \left(1 - \rho_1^k \cdot (k+1-k\cdot\rho_1)\right)}{\left[1+\rho_1^k \cdot (\rho - \rho_1 - \rho\cdot\rho_1)\right] \cdot (1-\rho_1)}.$$

According to the Little's formula (for instance, see (Kleinrock, 1975)) the residence time of messages in the communication link is defined by the relation

$$\bar{t}_l = \frac{\bar{r}}{\lambda} = \frac{\bar{r}}{\rho_1 \cdot \nu}.$$

Average residence time of messages in a network is

$$\bar{t}_r = \bar{t}_l + \bar{t}_{sv},$$

where the average servicing time of the message by the neuron-receiver $\bar{t}_{sv} = \frac{q}{\mu}, \ \ q = 1 - P_{k+1}$.

Then the average number of the messages connected with a network can be calculated by the following formula:

$$\bar{N} = \lambda \cdot \bar{t}_r.$$

## 3 Conclusion

Results of the executed research allow formulating following conclusions:

1. The area of usage of QS analytical models and their junction at study of neural networks is mainly limited due to the complexity of their decomposition, impossibility of the account of arbitrary distributions of a service time and entering streams of messages in each subsystem. Approximate solution of given problem is possible by means of the structurally functional approach and the method of stochastic normalization.

2. According to the method of normalization the elementary nervous network is represented in the form of normalized block of QS or for more complicated neural networks, representing connections of independent blocks incorporated by the general aim of service (Nikolsky, 2001), (Gamkrelidze & Nikolsky, 1993).

3. For definition of network operational characteristics for a stationary operating mode a priori knowledge concerning distribution laws is not required. Network characteristics are expressed by compact relations making them convenient for practical calculations.

## References

Brockman, J. (Eds.) (2002). *The Next Fifty Years: Science in the First Half of the Twenty-first Century.* New York, USA: Vintage Books, 320 p.

Gamkrelidze, E.L., Nikolsky, V.A. (1993). Construction and calculation of the normalized model of consecutive system of service. *Automatic Control and Computer Sciences*, 2, 36-40.

Kleinrock, L. (1975). *Queuing Systems. Vol. 1: Theory.* New York, USA: John Wiley & Sons, 417 p.

Nicholls, J.G., Martin,A.R., Wallace,B.G., Fuchs,P.A. (2001). *From Neuron to Brain.* New York, USA: Sinauer Associates, 679 p.

Nikolsky, V.A., Gamkrelidze, E.L. (1990), Construction and calculation of the basic performances of information systems by a normalization method. *Telecommunication*, 7, 21-22.

Nikolsky, V.A. (1990), Construction and calculation of the normalized model of the multiprocessing computing system with the General Memory. *Automatics and Computer Techniques*, 2, 62-66.

Virchow, R. (1854). Über das ausgebreitete Vorkommen einer dem Nervenmark analogen Substanz in den tierischen Geweben. *Archiv für pathologische Anatomie und Physiologie und für klinische Medicin*, 6(4), 562-572.